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Original article

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STATISTICAL SYNTHESIS OF A BAYESIAN ALGORITHM IMAGE SEGMENTATION AND MEASUREMENT OF AERIAL OBJECT COORDINATES

Abstract. This paper presents the results of a statistical synthesis of an algorithm for segmenting images of aerial objects based on the Bayesian criterion of maximum posterior probability. The key feature of the algorithm is the use of information about the operator's initial choice of the object to form a priori spatial distribution of coordinates, which allows effectively taking into account geometric constraints on the movement of the object between adjacent frames of the video sequence. A two-stage approach has been developed to jointly solve the tasks of pixel classification and object position estimation, in which spatial information is directly integrated into the segmentation decision rule through a Gaussian model of probability distribution. Analytical expressions for the optimal decision rule are obtained in the form of a threshold comparison of the log-likelihood ratio, which includes both intensity and spatial components. The resulting algorithm improves the quality of segmentation and the accuracy of coordinate measurements under varying lighting conditions, which is critically important for automatic tracking systems of aerial objects in the tasks of airspace monitoring and flight trajectory management.

Keywords: image segmentation, Bayesian algorithm, statistical synthesis, aerial objects, automatic tracking, spatial distribution, posterior probability, likelihood function, computer vision, interframe processing, state vector estimation, pixel classification

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СТАТИСТИЧЕСКИЙ СИНТЕЗ БАЙЕСОВСКОГО АЛГОРИТМА СЕГМЕНТАЦИИ ИЗОБРАЖЕНИЯ И ИЗМЕРЕНИЯ КООРДИНАТ ВОЗДУШНЫХ ОБЪЕКТОВ

Аннотация. Представлены результаты статистического синтеза алгоритма сегментации изображений воздушных объектов, основанного на байесовском критерии максимума апостериорной вероятности. Ключевой особенностью алгоритма является использование информации о начальном выборе объекта оператором для формирования априорного пространственного распределения координат, что позволяет эффективно учитывать геометрические ограничения на перемещение объекта между соседними кадрами видеопоследовательности. Разработан двухэтапный подход к решению задачи классификации пикселей и оценивания координат объекта, при котором пространственная информация интегрируется непосредственно в решающее правило сегментации через гауссову модель распределения вероятностей. Получены аналитические выражения для оптимального решающего правила в виде сравнения логарифма отношения правдоподобия, включающего яркостную и пространственную компоненты. Полученный алгоритм позволяет повысить качество сегментации и точность измерения координат в условиях изменяющегося освещения, что критически важно для систем автоматического сопровождения воздушных объектов в задачах мониторинга воздушного пространства и управления траекториями полета.

Ключевые слова: сегментация изображений, байесовский алгоритм, статистический синтез, воздушные объекты, автоматическое сопровождение, пространственное распределение, апостериорная вероятность, функция правдоподобия, компьютерное зрение, межкадровая обработка, оценка вектора состояния, классификация пикселей

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Introduction. Modern information technologies, in particular computer vision, play a key role in airspace monitoring tasks. The development of such fields as the defense industry, air traffic control systems, surveillance, and search-and-rescue operations demands the creation of highly efficient systems for automatic detection and tracking of aerial objects: fixed-wing aircraft, helicopters, unmanned aerial vehicles, and other airborne craft [1]. The task of automatic tracking of aerial objects comprises two primary stages: a single one-frame detection of the target by an operator on the first frame of the sequence, and automatic interframe tracking of the target in subsequent frames without operator involvement. The first stage is referred to as the one-frame processing (OFP) stage, and the second as the interframe processing (IFP) stage. A key requirement for successful interframe tracking is the reliable generation of single coordinate estimates (SCE) of the target's coordinates in each frame of the video sequence [2; 3]. The system must determine the current position of the target under varying external conditions: fluctuating illumination, changing observation angles, partial occlusion of the target by clouds or other objects, as well as the self-motion of the carrier platform of the optoelectronic system.

One approach to determining target coordinates involves solving the image segmentation problem followed by computing the centroid of the extracted segment. Segmentation enables accurate delineation of the target boundaries by separating the target from the background, thereby providing the basis for computing the target's center-of-gravity coordinates and generating control signals for the guidance system or flight trajectory tracking. Traditional segmentation methods, such as thresholding [4–8], gradient-based methods [9; 10], and clustering [11], frequently exhibit insufficient robustness under complex background conditions and varying observation environments. Modern approaches based on deep neural networks [12–16] require substantial computational resources and large volumes of training data, which may limit their applicability in real-time systems. A special place in the segmentation methodology is occupied by Bayesian approaches based on the statistical synthesis of optimal decision-making algorithms [17–21]. These approaches allow for a rigorous treatment of a priori uncertainties, effective utilization of the available operator-provided information, and adaptation to varying observation conditions. The practical challenge of implementing such algorithms lies in the fact that the statistical characteristics of target and background brightness values are a priori unknown and may vary considerably throughout the observation process.

This paper presents the results of the statistical synthesis of an image segmentation algorithm for aerial objects and the estimation of their coordinates for the purposes of automatic interframe tracking. The proposed algorithm is based on the Bayesian maximum a posteriori probability criterion, incorporating spatial information provided by the operator and a dedicated methodology for estimating brightness distribution parameters under conditions of a priori uncertainty. The issues of practical implementation of the derived algorithm, its experimental verification, and a comparative analysis of its performance relative to existing segmentation and coordinate estimation methods will be addressed in detail in subsequent publications.

Initial Data and Problem Statement. Consider a generated video sequence of the aerial target-background scene $\mathbf{f}'_k = \{\mathbf{f}_1, \mathbf{f}_2, ..., \mathbf{f}_k\}$ (Figure 1), which contains all observation-available information at the k-th frame, where $k = \overline{0,1,2..,K-1}$ is the frame index of the video sequence and K – is the total number of frames. By virtue of the Markov property of the target motion model, it is sufficient to use only the current observations $\mathbf{f}_k = \left\{f(x,y,k),(x,y)\in\Omega\right\}$, when solving the segmentation problem at the k-th frame, where $f(x, y, k)$ is the brightness of the pixel with coordinates (x, y), defining the known set of coordinates Ω of all pixels with a deterministic image structure.

At the first frame $k = 0$, the operator selects a rectangular region belonging to the object of interest. At this moment, two gates are formed: an inner gate and an outer gate [23]. The inner gate $\mathbf{S}^{\text{in}} = \left\| x_k^{\text{in}} \quad y_k^{\text{in}} \quad w_k^{\text{in}} \quad h_k^{\text{in}} \right\|^T$ characterizes the region in the image $\mathbf{f}_k$ belonging to the object of interest. The coordinates of its center are included in the observation vector $\boldsymbol{\theta}_k = \left\| x_k^{\text{in}} \quad y_k^{\text{in}} \right\|^T$ and are determined from the segmentation results at the k-th frame. The inner gate dimensions $\left(w_k^{\text{in}}, h_k^{\text{in}}\right)$ are set by the operator at the automated workstation according to the physical dimensions of the observed object. As a rule, the inner gate dimensions are chosen such that the object image fits within it with

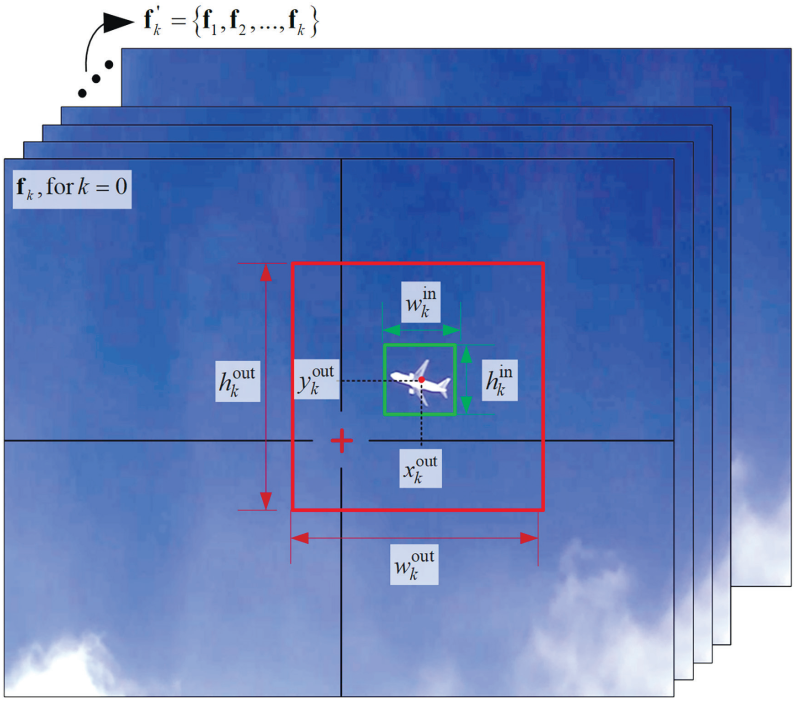

Figure 1. Sequence of discretized images of aerial background-target environment

a margin. The margin value should be selected based on the maximum possible change in object dimensions per frame [24].

The outer gate $\mathbf{S}^{\text{out}} = \left\| x_k^{\text{out}} \quad y_k^{\text{out}} \quad w_k^{\text{out}} \quad h_k^{\text{out}} \right\|^T$ represents the object search region in the image $\mathbf{f}_k$ and characterizes the dynamic parameters of the object. The coordinates of its center are included in the state vector $\boldsymbol{\alpha}_k = \left\| x_k^{\text{out}} \quad y_k^{\text{out}} \right\|^T$ and are formed on the basis of previous estimates and the dynamic model of the motion of the object. This provides protection against false alarms and abrupt changes in the gate center coordinates. The outer gate dimensions $\left(w_k^{\text{out}}, h_k^{\text{out}} \right)$ are selected based on the condition of a high probability that the coordinates of the object of interest fall within it, and may be adjusted at the interframe tracking stage [1].

In the object segmentation problem, each pixel of the image must be classified as belonging either to the object or to the background. To formalize this problem, two hypotheses are introduced: ϑ_0 – the hypothesis that the pixel belongs to the “background” class, and ϑ_1 – the hypothesis that the pixel belongs to the “object” class. It is assumed that the classification problem is solved independently at each pixel. The set of classification results for all pixels within the outer gate region forms the segmentation matrix $\mathbf{J} = \left\{ J(x, y), (x, y) \in \mathbf{S}^{\text{out}} \right\}$, where $J(x, y)$ – is the result of binary classification of the pixel with coordinates (x, y). The result $J(x, y) = 1$ corresponds to the acceptance of hypothesis ϑ_1, and $J(x, y) = 0$ – to the acceptance of hypothesis ϑ_0 respectively.

The maximum a posteriori probability criterion is chosen as the statistical synthesis criterion, which ensures the minimum Bayesian risk for a simple loss function [23]. Thus, the problem consists in synthesizing a device that jointly solves two tasks. The first task involves classifying all image pixels by choosing between the competing hypotheses of belonging to the object ϑ_1 and the background ϑ_0. The second task is aimed at estimating the state vector $\boldsymbol{\alpha}_k$ in the current frame taking into account previous measurements.

Statistical Synthesis of the Algorithm. Solving the segmentation and coordinate estimation problems requires the selection of an appropriate optimality criterion. The Bayesian approach provides the possibility of theoretically justified incorporation of prior information about the state distribution and observation uncertainties. The maximum a posteriori probability criterion ensures the minimum Bayesian risk for a simple loss function and is formulated as a joint optimization problem [20; 21]:

$$\left\{\hat{\mathbf{J}}, \hat{\boldsymbol{\alpha}}_k\right\} = \underset{\mathbf{J}, \boldsymbol{\alpha}_k}{\arg\max}\, p(\mathbf{J}, \boldsymbol{\alpha}_k \mid \mathbf{f}_k), \quad (1)$$

where $p(\mathbf{J}, \boldsymbol{\alpha}_k, | \mathbf{f}_k)$ – joint posterior probability density function (PDF) of the segmentation matrix and the state vector; $\hat{\mathbf{J}}$ – optimal segmentation matrix; $\hat{\boldsymbol{\alpha}}_k$ – optimal estimate of the state vector.

In order to ensure computational efficiency and applicability of the algorithm in real-time systems, the practical implementation employs a sequential optimization scheme, in which the segmentation problem is first solved based on the Bayesian decision rule, followed by the estimation of the object coordinates. This approach provides a balance between the rigor of the theoretical model and the processing speed requirements, while maintaining high object localization accuracy under conditions of limited contrast and complex background environments.

Applying Bayes' formula to the joint posterior PDF of the segmentation matrix and the state vector $p(\mathbf{J}, \boldsymbol{\alpha}_k, | \mathbf{f}_k)$ yields:

$$p\left(\mathbf{J}, \boldsymbol{\alpha}_k, | \mathbf{f}_k\right) = \frac{p\left(\mathbf{f}_k \mid \mathbf{J}, \boldsymbol{\alpha}_k\right) \cdot p\left(\mathbf{J}, \boldsymbol{\alpha}_k\right)}{p\left(\mathbf{f}_k\right)}, \quad (2)$$

where $p(\mathbf{f}_k \mid \mathbf{J}, \boldsymbol{\alpha}_k)$ – likelihood function (LF) of the pixel brightness observations conditioned on the segmentation matrix and the state vector; $p(\mathbf{J}, \boldsymbol{\alpha}_k)$ – joint prior PDF of the segmentation matrix and the state vector; $p(\mathbf{f}_k)$ – normalizing factor.

Since the normalizing factor $p(\mathbf{f}_k)$ does not affect the optimization result, criterion (2) simplifies to the following expression:

$$p(\mathbf{J}, \boldsymbol{\alpha}_k, | \mathbf{f}_k) = p(\mathbf{f}_k \mid \mathbf{J}, \boldsymbol{\alpha}_k) \cdot p(\mathbf{J}, \boldsymbol{\alpha}_k). \quad (3)$$

The key feature of the model is that the likelihood function of brightness observations $p(\mathbf{f}_k \mid \mathbf{J}, \boldsymbol{\alpha}_k)$ does not directly depend on the coordinates of the object center $\boldsymbol{\alpha}_k$, since the brightness of a pixel is determined solely by its belonging to either the object or background class. As a result, the likelihood function of brightness observations can be simplified to $p(\mathbf{f}_k \mid \mathbf{J}, \boldsymbol{\alpha}_k) = p(\mathbf{f}_k \mid \mathbf{J})$. Substituting this simplification into expression (3), we obtain:

$$p(\mathbf{J}, \boldsymbol{\alpha}_k \mid \mathbf{f}_k) = p(\mathbf{f}_{k+1} \mid \mathbf{J}) \cdot p(\mathbf{J}, \boldsymbol{\alpha}_k). \quad (4)$$

In practice, the likelihood function $p(\mathbf{f}_k \mid \mathbf{J})$ can be estimated by various methods, including nonparametric approaches using brightness histograms or parametric models with adaptation to varying illumination conditions [23; 24].

The prior PDF of the segmentation matrix and the state vector $p(\mathbf{J}, \boldsymbol{\alpha}_k)$ can be expressed as a product of the conditional distribution of the segmentation matrix given the object center coordinates and the prior PDF of the state vector:

$$p(\mathbf{J}, \boldsymbol{\alpha}_k) = P(\mathbf{J} \mid \boldsymbol{\alpha}_k) \cdot p(\boldsymbol{\alpha}_k), \quad (5)$$

where $P(\mathbf{J} \mid \boldsymbol{\alpha}_k)$ – conditional probability of the segmentation matrix given the object center coordinates; $p(\boldsymbol{\alpha}_k)$ – prior PDF of the state vector containing the object coordinates.

To describe the conditional probability of the segmentation matrix $P(\mathbf{J} \mid \boldsymbol{\alpha}_k)$ given the state vector, a model is employed that accounts for the spatial structure of the image and assumes statistical independence of decisions across individual pixels:

$$P(\mathbf{J} \mid \boldsymbol{\alpha}_k) = \prod_{(x,y) \in \mathbf{S}^{\text{out}}} P(J(x,y) \mid \boldsymbol{\alpha}_k, x, y), \quad (6)$$

where $P(J(x,y) \mid \boldsymbol{\alpha}_k, x, y)$ – conditional probability of classifying the pixel with coordinates (x, y) given the state vector $\boldsymbol{\alpha}_k$.

As the state vector at the segmentation stage, its extrapolated estimate $\boldsymbol{\alpha}_k = \hat{\boldsymbol{\alpha}}_{0k}$, $\hat{\boldsymbol{\alpha}}_{0k} = \left\| \hat{x}_{0k} \quad \hat{y}_{0k} \right\|^T$ is used, which is formed at the IFP stage based on a dynamic model of object motion (e.g., the constant velocity model), taking into account previous estimates of the coordinates and their accuracy, as well as the noise of the motion model and measurements [25]:

$$\hat{\boldsymbol{\alpha}}_{0k} = \mathbf{B}\hat{\boldsymbol{\alpha}}_{k-1}, \tag{7}$$

where $\mathbf{B}$ – deterministic dynamic state transition matrix that recalculates the state vector increments from the $(k-1)$-th to the k-th frame, whose structure is determined at the IFP stage; $\hat{\boldsymbol{\alpha}}_{k-1}$ – is the state vector estimate at the $(k-1)$-th frame.

Thus, for the hypothesis ϑ_1 the conditional probability of pixel classification $P(J(x,y) = \vartheta_1 \,|\, \boldsymbol{\alpha}_k = \hat{\boldsymbol{\alpha}}_{0k}, x, y)$ is described by the expression:

$$P(\vartheta_1 \,|\, \hat{\boldsymbol{\alpha}}_{0k}, x, y) \approx \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\frac{\left(x-\hat{x}_{0k}\right)^2}{\sigma_x^2} + \frac{\left(y-\hat{y}_{0k}\right)^2}{\sigma_y^2}\right]\right\}, \tag{8}$$

where σ_x, σ_y standard deviations (SD) of the Gaussian model of the spatial probability distribution of object membership along the x and y coordinates, respectively.

The approximate equality of the Gaussian model in expression (8) is explained by the fact that pixel coordinates are discrete, whereas the Gaussian model is continuous, and when σ_x, $\sigma_y \gg 1$, the value of $P(\vartheta_1 \,|\hat{\boldsymbol{\alpha}}_{0k}, x, y)$ can be used as an approximation to the probability of a pixel belonging to the object.

In accordance with the three-sigma rule [21], the SD values σ_x, σ_y are determined by the following relations:

$$\sigma_x = \frac{w_k^{\text{in}}}{6}, \quad \sigma_y = \frac{h_k^{\text{in}}}{6}. \tag{9}$$

Since the shape of the target image is not defined in advance and can have an arbitrary form, the prior probability density function (PDF) of the state vector under the condition of target absence ϑ_0 obeys a uniform distribution:

$$P\left(\vartheta_0 \,|\, \hat{\boldsymbol{\alpha}}_{0k}, x, y\right) = \frac{1}{h_k^{\text{in}} w_k^{\text{in}}}.$$

The prior information about the location of the object is formed at the IFP stage based on a dynamic model of object motion and previous coordinate estimates. In this work, it is assumed that the Gaussian PDF of the extrapolated state vector estimate is used as prior information (Figure 2):

$$p(\boldsymbol{\alpha}_k = \hat{\boldsymbol{\alpha}}_{0k}) = \left|2\pi\mathbf{R}_{0k}\right|^{-1/2} \exp\left\{-\frac{1}{2}(\hat{\boldsymbol{\alpha}}_{0k} - \hat{\boldsymbol{\alpha}}_k)^T \mathbf{R}_{0k}^{-1}(\hat{\boldsymbol{\alpha}}_{0k} - \hat{\boldsymbol{\alpha}}_k)\right\}, \tag{10}$$

where $\hat{\boldsymbol{\alpha}}_k$ – extrapolated state vector estimate at the k-th frame; $\mathbf{R}_{0k} = \begin{Vmatrix} \sigma_{0x}^2 & 0 \\ 0 & \sigma_{0y}^2 \end{Vmatrix}$ – diagonal covariance matrix of the state vector extrapolation errors with elements σ_{0x}^2, σ_{0y}^2.

Thus, the proposed approach to object segmentation allows implementing a dual-function decision rule that performs optimization sequentially in two stages. The first stage consists in determining the optimal segmentation matrix $\hat{\mathbf{J}}$ for a fixed state vector $\boldsymbol{\alpha}_k = \hat{\boldsymbol{\alpha}}_{0k}$. The second stage consists in finding the optimal state vector $\hat{\boldsymbol{\alpha}}_k$ for a known segmentation matrix $\hat{\mathbf{J}}$.

For a fixed state vector, the prior PDF of the object coordinates becomes a constant $p(\boldsymbol{\alpha}_k) = \text{const}$ and does not affect the optimization result with respect to the segmentation matrix $\mathbf{J}$. This problem

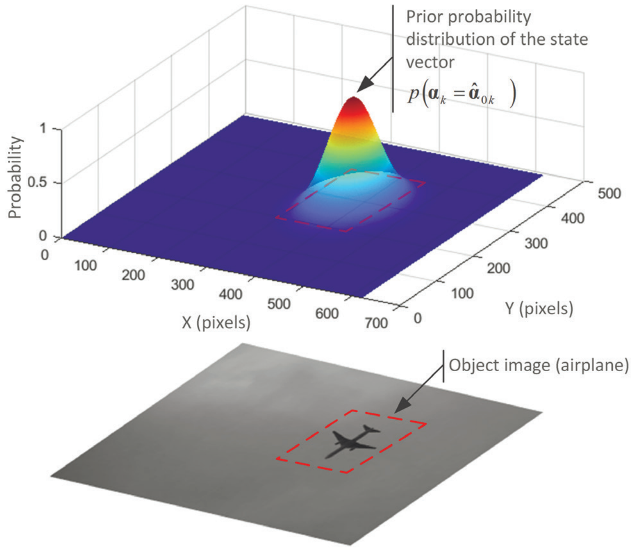

Figure 2. Image of aerial background-target environment with Gaussian probability density of observed object coordinates

corresponds to the use of a simple loss function [21], in which the misclassification error of each pixel has equal weight. Furthermore, the classification problem reduces to independently solving hypothesis testing problems for each pixel:

$$\hat{J}(x,y) = \arg\max p(f(x,y,k) \mid J(x,y)) \cdot P(J(x,y) \mid \boldsymbol{\alpha}_k, x, y), \tag{11}$$

where $p(f(x,y,k) \mid J(x,y))$ – values of the conditional PDF $p(\mathbf{f}_{k+1} \mid \mathbf{J})$ at the point with coordinates (x, y) under the acceptance of hypothesis $J(x, y)$.

Applying criterion (11) to an individual pixel leads to the rule of selecting the hypothesis with the higher posterior probability. In practice, the LF $p(f(x,y,k) \mid J(x,y))$ is not available in explicit form [25]. Instead, either the likelihood ratio $\Lambda(x, y)$, or a monotonic transformation of it is formed:

$$\Lambda(x,y) = \frac{p\left(f(x,y,k) \mid \vartheta_1\right) \cdot P\left(\vartheta_1 \mid \boldsymbol{\alpha}_k, x, y\right)}{p\left(f(x,y,k) \mid \vartheta_0\right) \cdot P\left(\vartheta_0 \mid \boldsymbol{\alpha}_k, x, y\right)}. \tag{12}$$

The most common transformation is the log-likelihood ratio $\ln\Lambda(x, y)$, which ensures computational stability and simplification of arithmetic operations [24]:

$$\ln\Lambda(x,y) = \underbrace{\ln\left(\frac{p\left(f(x,y,k) \mid \vartheta_1\right)}{p\left(f(x,y,k) \mid \vartheta_0\right)}\right)}_{\text{Brightness component}} + \underbrace{\ln\left(\frac{P\left(\vartheta_1 \mid \boldsymbol{\alpha}_k, x, y\right)}{P\left(\vartheta_0 \mid \boldsymbol{\alpha}_k, x, y\right)}\right)}_{\text{Spatial component}}. \tag{13}$$

The decision rule for an individual pixel is formulated based on the log-likelihood ratio (13):

$$\hat{J}(x,y) = \begin{cases} 1, \text{ if } \ln\Lambda(x,y) > 0 \\ 0, \text{ if } \ln\Lambda(x,y) \le 0 \end{cases}. \tag{14}$$

Expression (14) contains two components, each of which contributes to the decision-making process. The brightness component reflects the differences in the brightness distributions of the object and the background based on the gates. The spatial component accounts for the position of the pixel relative to the presumed center of the object, where the parameters σ_x and σ_y are associated with the fixed dimensions of the inner gate $\mathbf{S}^{\text{in}}$. Applying the decision rule (14) to all pixels of the outer gate region $\mathbf{S}^{\text{out}}$ yields the optimal estimate of the segmentation matrix $\hat{\mathbf{J}}$, where the decision for each pixel is made independently based on the analysis of the two aforementioned components.

The second stage of synthesis is aimed at determining the optimal state vector for a known segmentation matrix $\hat{\mathbf{J}}$, obtained at the first stage. Since the LF $p(\mathbf{f}_k \mid \hat{\mathbf{J}})$ does not depend on the value of $\boldsymbol{\alpha}_k$, it is excluded from the optimization expression (4). Thus, for a fixed matrix $\hat{\mathbf{J}}$ the problem reduces to maximizing the criterion with respect to the state vector only:

$$\hat{\boldsymbol{\alpha}}_k = \arg\max_{\boldsymbol{\alpha}_k} P(\hat{\mathbf{J}} \mid \boldsymbol{\alpha}_k) \cdot p(\boldsymbol{\alpha}_k). \tag{15}$$

In practice, for the measurement problem it is convenient to transition to an equivalent minimization problem by introducing the negative logarithm [21]:

$$\hat{\boldsymbol{\alpha}}_k = \arg\min_{\boldsymbol{\alpha}_k} \Pi(\boldsymbol{\alpha}_k), \tag{16}$$

where $\Pi(\boldsymbol{\alpha}_k) = -\ln P(\hat{\mathbf{J}} \mid \boldsymbol{\alpha}_k) - \ln p(\boldsymbol{\alpha}_k)$ – loss function.

Thus, the problem of likelihood maximization transitions into the problem of minimizing the loss function $\Pi(\boldsymbol{\alpha}_k)$.In this case, likelihood maximization and loss function minimization are mutually inverse operations [21; 22]:

Substituting expressions (8) and (10) into (16), we obtain:

$$\Pi(\boldsymbol{\alpha}_k) = \sum_{\hat{J}(x,y)=1} \left[\frac{\left(x - \hat{x}_{0k}\right)^2}{\sigma_x^2} + \frac{\left(y - \hat{y}_{0k}\right)^2}{\sigma_y^2} \right] + \frac{\left(\hat{x}_{0k} - \hat{x}_k^{\text{out}}\right)^2}{\sigma_{0x}^2} + \frac{\left(\hat{y}_{0k} - \hat{y}_k^{\text{out}}\right)^2}{\sigma_{0y}^2}. \tag{17}$$

The loss function (17) represents a weighted sum of squared deviations, which is a standard quadratic form for Bayesian estimation problems under Gaussian prior distributions of the object center coordinates. The first term characterizes the correspondence of the object center coordinate estimate to the pixels classified as belonging to the object, with weights inversely proportional to the corresponding variances of the spatial model. The second term provides regularization through prior information from the Kalman filter.

The coordinates of the center of mass of the segmented region are determined by the following expression:

$$\hat{x}_k^{\text{in}} = \frac{1}{N} \sum_{\hat{J}(x,y)=1} x, \quad \hat{y}_k^{\text{in}} = \frac{1}{N} \sum_{\hat{J}(x,y)=1} y, \tag{18}$$

where N – number of object pixels.

By transforming expression (17) taking into account (18) and computing the partial derivatives with respect to the object center coordinates and setting them to zero, we obtain:

$$\frac{\partial}{\partial \hat{x}_k^{\text{out}}} = \frac{N\left(\hat{x}_k^{\text{in}} - \hat{x}_{0k}\right)}{\sigma_x^2} + \frac{\left(\hat{x}_k^{\text{out}} - \hat{x}_{0k}\right)}{\sigma_{0x}^2} = 0, \quad \frac{\partial}{\partial \hat{y}_k^{\text{out}}} = \frac{N\left(\hat{y}_k^{\text{in}} - \hat{y}_{0k}\right)}{\sigma_y^2} + \frac{\left(\hat{y}_k^{\text{out}} - \hat{y}_{0k}\right)}{\sigma_{0y}^2} = 0. \tag{19}$$

Solving equations (19) with respect to the object coordinate estimates $\hat{x}_k^{\text{out}}$, $\hat{y}_k^{\text{out}}$ the expression for calculating the state vector estimate takes the form:

$$\hat{\boldsymbol{\alpha}}_k = \hat{\boldsymbol{\alpha}}_{0k} + \mathbf{K}\left(\hat{\boldsymbol{\theta}}_k - \mathbf{H}\hat{\boldsymbol{\alpha}}_{0k}\right), \tag{20}$$

where $\hat{\boldsymbol{\alpha}}_k = \left\| \hat{x}_k^{\text{out}} \quad \hat{y}_k^{\text{out}} \right\|^T$ – state vector estimate containing the object coordinate estimates; $\hat{\boldsymbol{\theta}}_k = \left\| \hat{x}_k^{\text{in}} \quad \hat{y}_k^{\text{in}} \right\|$ – observation vector estimate containing the one-time estimates of the center of mass coordinates of the segmented region; $\mathbf{H}$ – identity static recalculation matrix of state vector changes into observation vector changes; $\mathbf{K} = \begin{Vmatrix} K_x & 0 \\ 0 & K_y \end{Vmatrix}$ – gain coefficient matrix with elements $K_x = \dfrac{N\sigma_{0x}^2}{N\sigma_{0x}^2 + \sigma_x^2}$, $K_y = \dfrac{N\sigma_{0y}^2}{N\sigma_{0y}^2 + \sigma_y^2}$.

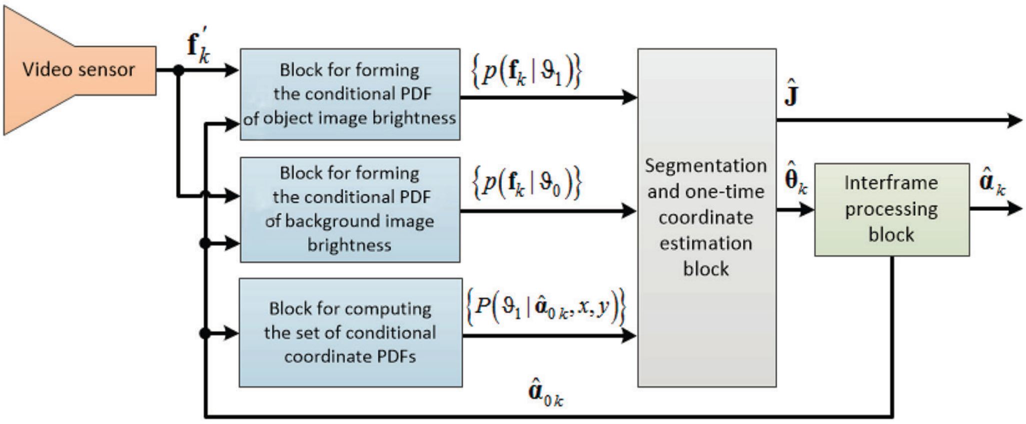

Figure 3. Structural diagram of the algorithm for joint segmentation and coordinate estimation of the object

As a result of statistical synthesis, the structure of a device for the joint solution of image segmentation and aerial object coordinate estimation problems has been obtained (Figure 3). The algorithm processes the video sequence frame by frame. The input data for processing the k-th frame is the extrapolated state vector estimate $\hat{\boldsymbol{\alpha}}_{0k}$ from the IFP block.

The extrapolated estimate $\hat{\boldsymbol{\alpha}}_{0k}$ is used to set the positions of gates $\mathbf{S}^{\text{in}}$ and $\mathbf{S}^{\text{out}}$ on the current image frame $\mathbf{f}_k$. Further processing of the video sequence is carried out by three main blocks. The block for forming the conditional PDF of the object image brightness computes the value of $p(\mathbf{f}_k \mid \vartheta_1)$ from the set of pixels within the inner gate, whose center is aligned with $\hat{\boldsymbol{\alpha}}_{0k}$. Similarly, the block for forming the conditional PDF of the background image brightness estimates the distribution parameters of $p(\mathbf{f}_k \mid \vartheta_0)$ from the pixel sample in the annular region between the boundaries of the outer and inner gates $\mathbf{S}^{\text{out}}$ and $\mathbf{S}^{\text{in}}$ respectively. Simultaneously, the block for computing the conditional coordinate PDFs forms the set of probabilities $P(\vartheta_1 \mid \hat{\boldsymbol{\alpha}}_{0k}, x, y)$ for each pixel of the search region based on the two-dimensional Gaussian function (8).

The outputs of all three blocks are fed into the segmentation and one-time coordinate estimation block. This block implements the decision rule (14), which for each pixel of the outer gate region $\mathbf{S}^{\text{out}}$ computes the log-likelihood ratio by combining the brightness and spatial components. Application of the decision rule (14) results in the formation of the binary segmentation matrix $\hat{\mathbf{J}}$. Based on the obtained matrix, the center of mass coordinates of the segmented region $\hat{\boldsymbol{\theta}}_k$ are computed, which together with the extrapolated estimate $\hat{\boldsymbol{\alpha}}_{0k}$ and the gain matrix $\mathbf{R}_{0k}$ are used to compute the final state vector estimate $\hat{\boldsymbol{\alpha}}_k$ in accordance with expression (20). This extrapolated estimate is fed into the interframe processing block to initialize the computational cycle for the next frame. The presented structure implements a closed-loop recursive information processing scheme with prediction and correction of estimates, which ensures adaptation to object motion and enables accounting for the spatiotemporal relationship between adjacent frames of the video sequence.

Conclusion. The presented algorithm for statistical synthesis of joint segmentation and coordinate measurement of aerial objects provides a theoretically justified solution to the automatic tracking problem based on the maximum a posteriori probability criterion. The key feature of the developed approach is the joint solution of pixel classification and coordinate measurement problems, taking into account their interrelation through spatial information, which outperforms traditional sequential processing methods in terms of overall solution quality and ensures optimal utilization of all available information about the geometric structure of the object. The synthesized decision rule can be effectively applied in automatic tracking systems for aerial objects of various types, ensuring robust determination of their coordinates under challenging observation conditions and opening prospects for further development of adaptive segmentation methods. The issues of software implementation of the developed algorithm, experimental investigation of its characteristics on real video sequences, and comparative analysis of its effectiveness relative to existing segmentation and coordinate measurement methods will be presented in subsequent publications.

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